Int. J. Heat Mass Transfer. Vol. 6, pp. 853–862. Pergamon Press 1963. Printed in Great Britain.

THE INFLUENCE OF ELECTROSTRICTIVE FORCES IN NATURAL THERMAL CONVECTION

P. S. LYKOUDIS and C. P. YU

School of Aeronautical and Engineering Sciences, Purdue University

(*Received* 4 *October* 1962 *and in revised form* 18 *January* 1963)

Abstract—The free convection of an electrostrictive fluid enclosed between a horizontally heated wire and a coaxial, cooled cylinder, under an applied nonuniform radial electric field, is examined. An approximate solution is obtained by extending Langmuir's conduction model for fine wires to the electrostrictive case. It is found that the increase in Nusselt number due to the additional circulation produced by the electrostrictive force depends on the Grashof number, Prandtl number and a third characteristic non-dimensional number of the same nature as the Grashof number but based on an electrostatic rather than gravitational field. The theoretical results are compared with available experimental data and are found in good agreement.

NOMENCLATURE

$\vec{B}$, magnetic flux;
$\vec{D}$, electric displacement vector;
d, diameter of the wire;
$\vec{E}$, electric field;
E_s, electric field at surface of the wire;
$\vec{f}$, force per unit volume on the fluid;
$\vec{f}_0$, force in the absence of heating;
Gr, Grashof number, $\dfrac{\beta g \theta_s d^3}{\nu^2}$;
$\vec{g}$, gravitational acceleration;
$\mathbf{g}^*$, equivalent gravitational acceleration for the forces of electrostriction;
$\vec{H}$, magnetic field intensity;
k, Boltzmann's constant;
M, molecular weight of the gas molecule;
N, Avogadro's number;
Nu, Nusselt number, $\dfrac{Q}{\pi \lambda \theta_s}$;
ΔNu, change of Nu due to the electric field;
Pr, Prandtl number, $\dfrac{\nu}{\alpha}$;
p, pressure in the fluid;
p_0, permanent electric moment of a gas molecule;
Q, heat transferred per unit length per unit time;
$\vec{q}$, velocity of the fluid;
r, radius of the wire;
U, internal energy;
Se, Senftleben number $\dfrac{\beta \gamma \theta_s E_s^2 d^2}{\nu^2}$;
T, absolute temperature of the fluid;
T_0, absolute temperature at the surface of the cylinder;
u, velocity component in vertical direction.

Greek symbols
α, thermal diffusivity;
β, coefficient of thermal expansion, $\dfrac{1}{T_0}$;
γ, constant, specifying the temperature dependence of χ defined by (A-6) and (A-7);
δ, idealized boundary layer thickness;
ϵ, dielectric constant;
θ, temperature difference;
θ_s, temperature difference at the surface of the wire;
λ, coefficient of heat conduction of the fluid;
μ_e, magnetic permeability;

ν, kinematic viscosity of the fluid;
ρ, mass density;
ρ_0, mass density in the absence of heating;
σ, coefficient of electric polarization;
χ, electric susceptibility of the fluid;
χ_0, electric susceptibility in the absence of heating.

INTRODUCTION

FROM the classical theory of electricity and magnetism we know that the "Maxwell stresses" give rise to body forces made up of the following components [1]: electrostatic (applied on free electric charges); ponderomotive (the macroscopic summation of the elementary Lorentz forces applied on charged particles); electrostrictive (present when the dielectric constant is a function of the mass density); a force due to an inhomogeneous dielectric; its magnetic counterpart, and the magnetostrictive force which is present when the magnetic permeability is a function of the mass density.

For an electrically neutral but electrically conducting fluid in the presence of an electromagnetic field the only substantial force in a great number of applications is the ponderomotive force. Indeed, what is called today magneto-fluid-mechanics deals exclusively with this force. However there are cases in which the forces associated with the dielectric constant, although normally small, become important when compared with equally small forces such as bouyant ones. This paper deals with such a case.

About 1930 Senftleben and Braun [2] discovered that when an electric field is applied radially between a horizontal heated wire and a cylinder, the heat-transfer rate increases up to 50 per cent provided a paraelectric gas (a gas whose molecules carry a permanent electric moment), is used. The authors attributed this increase to the additional thermal circulation current produced by the electrostrictive forces. Fig. 1 shows the general layout of this experiment.

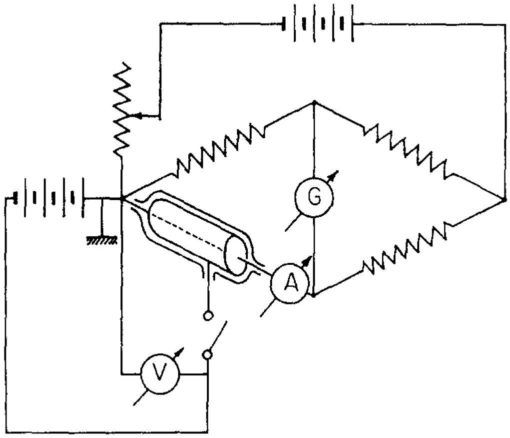

FIG. 1. Experimental layout used by Senftleben and Braun [2] for the study of the influence of electrostrictive forces in natural thermal convection.

According to Debye's theory of electric polarization, variable electric susceptibility of gases arises from the following two causes: (a) the presence of an electric field which, even in a symmetrical molecule, will induce a dipole moment (electric polarization) and (b) if the dipole possesses a permanent dipole moment (non-symmetric molecules), the molecule will tend to align itself with the electric field. The electric susceptibility due to the first cause is proportional to the mass density and inversely proportional to the absolute temperature [3]. As a result of the above dependence, if a cold volume element is found in a nonuniform electric field, the force at this point will exceed the force to which a hot volume in the same position is subjected. A tendency is thus produced for the cold volume elements to replace the hot ones as in the case of a pure gravitational force.

Kronig and Schwarz [4] adopted this physical model and forwarded some arguments based on general nondimensional considerations. They employed the usual definitions of Nusselt number (Nu), Grashof number (Gr) and Prandtl number (Pr) as well as a new nondimensional number of the same nature as the Grashof number, but based on an electrostatic rather than gravitational field. In this work we call this number Senftleben number (Se)† from the name of one of the research workers who we believe first studied this effect [2].

Kronig and Schwarz [4] assumed that the Nusselt number can be written in the following functional form‡

† In [4] this number is called "electrical characteristic number" and abbreviated to El.
‡ This form is suggested from experiments.

$$Nu = F(Gr \cdot Pr) + G(Se \cdot Pr). \quad (1)$$

The change of Nusselt number due to the application of the electric field is then given by:

$$\Delta Nu = G(Se \cdot Pr). \quad (2)$$

Senftleben's results on gases [1] appear to agree with this dependence satisfactorily, if the physical constants are based on the temperature of the wire, since all the results can be collected into a narrow band (an almost single curve) by using (ΔNu) and ($Se \cdot Pr$) as co-ordinates. In order to further confirm the above arguments, Ashmann and Kronig [5] and De Haan [6] conducted several experiments with polar liquids. They found that when an electric field is applied, the heat transfer increases generally in the same fashion as in gases.

Our purpose in the present paper is to forward further the non-dimensional analysis of [4] by providing an analytic expression for the Nusselt number in its dependence on the Grashof, Prandtl and Senftleben number. This is done only in an approximate way, since the exact solution of the conservation equations is extremely difficult (if at all possible), because, apart from the fact that the equations are coupled, it is not possible to reduce them to ordinary differential equations through similarity arguments, nor are they of the boundary layer type.† The approximation is based on an extension of Langmuir's idea [7] of an effective "conduction ring".

2. ANALYSIS

The problem of free convection of a heated fine wire with an electric field has been considered by Langmuir [7]; he idealized the boundary layer by a stationary cylindrical ring around the wire, through which the heat is transferred from the wire by the mechanism of thermal conduction only. If the idealized equivalent "conduction ring" thickness is δ, the Nusselt number (defined explicitly in the list of symbols) is given by

$$Nu = \frac{2}{\ln\,[1 + (2\delta/d)]} \quad (3)$$

† Because the boundary layer thickness for a very thin wire is of the same order of magnitude or greater than the radius of the wire.

For "thick" wires we have $2\delta/d \ll 1$; for very fine wires, under investigation here, we have $2\delta/d \gg 1$. In the case of free convection with no electric field, $2\delta/d$ is determined by considering the average film heat-transfer coefficient for the wire having the same value as the average film heat-transfer coefficient on a vertical plate with a height‡ of $2{\cdot}5d$; the results are

$$\frac{2\delta}{d} = \left(\frac{546}{Gr \cdot Pr}\right)^{1/4} \quad \text{for gases} \quad (4a)$$

$$\frac{2\delta}{d} = \left(\frac{219}{Gr \cdot Pr}\right)^{1/4} \quad \text{for liquids§}. \quad (4b)$$

The Grashof and Prandtl numbers are defined explicitly in the list of symbols. Substituting these expressions for δ/d into equation (3) gives

$$Nu = \frac{2}{\ln\left[1 + \left(\frac{546}{Gr \cdot Pr}\right)^{1/4}\right]} \quad \text{for gases} \quad (5a)$$

$$Nu = \frac{2}{\ln\left[1 + \left(\frac{219}{Gr \cdot Pr}\right)^{1/4}\right]} \quad \text{for liquids.} \quad (5b)$$

In extending these solutions to the case where an electric field is present, let us consider the fundamental conservation equations after the following assumptions are made:

1. The flow is steady.
2. For a very long horizontal wire, the problem is a two dimensional one; end effects are not considered.
3. The flow is assumed to be semi-incompressible, with the viscosity and thermal conductivity remaining constant.

The conservation equations of mass, momentum and energy are then as follows:

$$\nabla \cdot \vec{q} = 0 \quad (6)$$

‡ The numerical value of $2{\cdot}5$ is suggested by Hermann's solution [8] for a finite cylinder where the boundary layer assumption is true.

§ 546 and 219 are obtained from Eckert's solution over a vertical plate [9]. We have also used $Pr = 0{\cdot}75$ for gases and $Pr = 5{\cdot}5$ for liquids in the calculations of these constants.

$$\vec{q} \cdot \nabla \vec{q} = -\frac{\nabla p}{\rho} - \vec{g}\beta\theta - \tfrac{1}{2}\beta\gamma\theta\nabla E^2 + \nu\nabla^2 q \quad (7)$$

$$\vec{q} \cdot \nabla\theta = \alpha\nabla^2\theta. \quad (8)$$

In the above θ is the temperature difference of the fluid relative to the temperature of the cylinder. In (7), $-\vec{g}\beta\theta$ is the usual force term in free convection due to gravity, the electrostrictive force is given by† $-\frac{1}{2}\beta\gamma\theta\nabla E^2$. In (8), we have neglected the viscous dissipation and the part of the internal energy due to polarization.‡

The mechanism of action of the electrostrictive forces is depicted in Fig. 2. The points A, A', B and B' are the corners of a rectangle with the horizontally heated wire located at its center. The electrostrictive forces act in the outward radial direction as shown in the figure, with the forces at the points A and A' being larger than the ones acting at B and B', because a higher temperature isotherm passes through them. Furthermore we note that because of the symmetry of the temperature field with respect to the vertical plane passing through the axis of the wire, the horizontal components of the electrostrictive forces will balance each other.

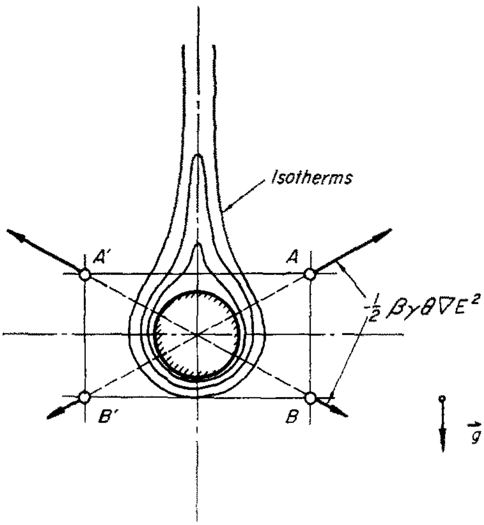

FIG. 2. The mechanism of action of electrostriction in the presence of natural thermal convection.

It follows from the above discussion that the net amount of the electrostrictive forces is in the upward direction and their order of magnitude from (7) is $(\beta\gamma\theta_s E^2)/(r + \delta)$. On the other hand, from the fact that $\nabla \cdot \vec{E} = 0$ and for very thin wires we have $E/E_s = r/(r + \delta) \simeq r/\delta$ and hence the force is equal to $\beta\gamma\theta_s E_s^2 r^2/\delta^3$. At this point we assume, as in the case of free convection from a fine wire, that the velocities are so small that the inertia forces terms involving the square of the velocities may be neglected. Moreover, since there is no pressure difference imposed from the outside, the pressure force in the vertical direction is negligible as in the case of the free convection from a vertical plate.

Bearing in mind the above, the order of magnitude of the vertical component of the momentum equation and the energy equation may be written in the following average sense:

$$C_1 g\beta\theta_s + C_2 \frac{\beta\gamma\theta_s E_s^2 d^2}{\delta^3} = \nu \frac{u}{\delta^2} \quad (9)$$

$$C_3 u \frac{\theta_s}{d} = \alpha \frac{\theta_s}{\delta^2} \quad (10)$$

where u is the velocity component in the vertical direction and C_1, C_2, C_3 are appropriate constants. Eliminating the velocity u from (9) and (10), we obtain after some rearrangement

$$\left(\frac{2\delta}{d}\right)^4 + C_4 \frac{Se}{Gr} \left(\frac{2\delta}{d}\right) = \frac{C_5}{Gr \,.\, Pr} \quad (11)$$

where

$$C_4 = 8\frac{C_2}{C_1}, \quad C_5 = \frac{16}{C_2 C_3} \quad (12)$$

and

$$Se = \frac{\beta\gamma\theta_s E_s^2 d^2}{\nu^2}. \quad (13)§$$

In the case of no electric field, $Se = 0$, and the solution of (11) for $2\delta/d$ is:

$$\frac{2\delta}{d} = \left(\frac{C_5}{Gr \cdot Pr}\right)^{1/4}. \quad (14)$$

† The derivation is given in Appendix A.

‡ See Appendix B for a complete discussion of the energy equation.

§ Comparing this definition with the definition for the Grashof number we see that they are the same if an equivalent gravitational field $\mathbf{g}^*$ is chosen as follows $\mathbf{g}^* = \gamma E_s{}^2/d$.

Comparing (14) with (5), we have $C_5 = 546$ for gases, and $C_5 = 219$ for liquids.

When Se is not zero, we must solve (11) for $2\delta/d$ and substitute into (3) to find Nu as a function of the quantities $Gr \cdot Pr$ and $Se \cdot Pr$. The result is rather cumbersome and hence we prefer to present the ratio $2\delta/d$ as a function of Nu as follows:

$$\frac{2\delta}{d} = e^{2/Nu} - 1. \tag{15}$$

Substituting (15) into (11) and rearranging yields

$$Se \cdot Pr = \frac{C_5 - Gr \cdot Pr\,(e^{2/Nu} - 1)^4}{C_4\,(e^{2/Nu} - 1)}. \tag{16}$$

The constant C_4 in (16) will be determined from experiments and it is evident that it should depend on the geometry of the experiment and at most the Prandtl number of the fluid.†

For very large values of Se, it is seen in (16) that the quantity $(Gr \cdot Pr)\,[(e^{2/Nu}) - 1]^4$ becomes negligible and $Nu \simeq \Delta Nu$. Equation (16) then takes the following asymptotic form

$$(Se \cdot Pr) \simeq \frac{C_5}{C_4\,(e^{2/\Delta Nu} - 1)}$$

or

$$\Delta Nu \simeq \frac{2C_4}{C_5}\,(Se \cdot Pr). \tag{17}$$

For a given geometry, equation (17) is the same as the one given by Kronig and Schwarz [4] if the function G is linear. From above we see that this result is valid only for very high $(Se \cdot Pr)$ numbers, as one would expect on physical grounds, since this is the case when the viscous force balances primarily the electrostrictive force while all other forces are negligible.

In Fig. 3 we plot the Nusselt number versus the product $(Se \cdot Pr)$ for different values of $(Gr \cdot Pr)$ based on the calculations of equation (16), where the constants C_4 and C_5 are taken to be 5·56 and 546 respectively.‡ It is seen that for small $(Se \cdot Pr)$ the convection is due almost entirely to the gravitational force, however as $(Se \cdot Pr)$ increases the electrostrictive convection becomes important and finally dominates the convective heat transfer.

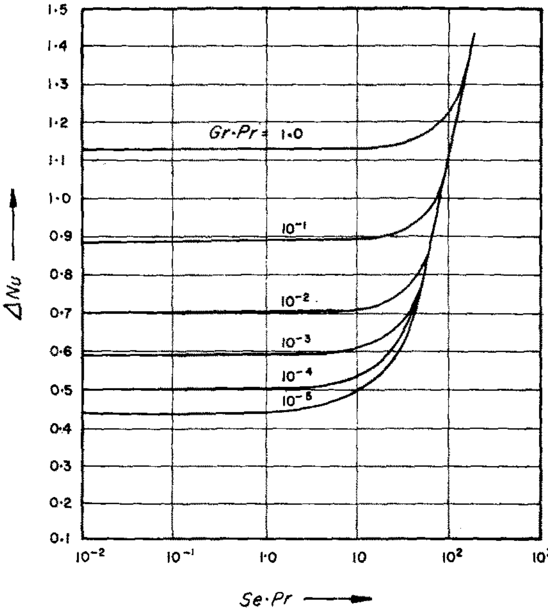

FIG. 3. The theoretical result yielding the Nusselt number versus the product $(Se \cdot Pr)$ for different combinations of the product $(Gr \cdot Pr)$ for gases.

3. COMPARISON WITH EXPERIMENTS

Senftleben and Braun [2] have measured the difference in heat transfer per unit time due to an electric field for a platinum wire of 0·03 mm in diameter and 7 cm in length placed at the center of a cylinder 34 mm in diameter. The cylinder was kept at temperature between 90° and 400°K. The following gases were tested: argon, oxygen, and ethyl chloride at pressures ranging between 87 and 740 mm Hg. The temperature differences between wire and cylinder were 41° and 82° and the electric field strength at the surface of the wire varied between 42 and 108 KV/cm. Ashmann and Kronig [5] performed similar measurements for the following liquids: toluene, n-heptane, n-hexane and carbon tetrachloride. A platinum wire of 20 μm in diameter and 7 cm long was placed inside a 4 cm diameter cylinder the surface of which was

† This dependence is revealed in the ordinary natural convection case when we take into account the inertia. See the footnote corresponding to equation (4).

‡ It will be seen later that these values correspond to gases.

kept at 0°C temperature. The temperature difference between the wire and the cylinder varied from 36° to 78° and the applied voltage between the cylinder and the wire from 113 to 561 V. Measurements on the same liquids were also made by De Haan [6] with a wire of 16 cm long under slightly different temperatures and applied voltages. In these last experiments the diameter of the wire was varied.

All thermophysical properties in the above experiments have been calculated at the wire-temperature. Fig. 4 shows a comparison between equation (16) and Senftleben's experiment in gases for $C_4 = 5{\cdot}56$.†

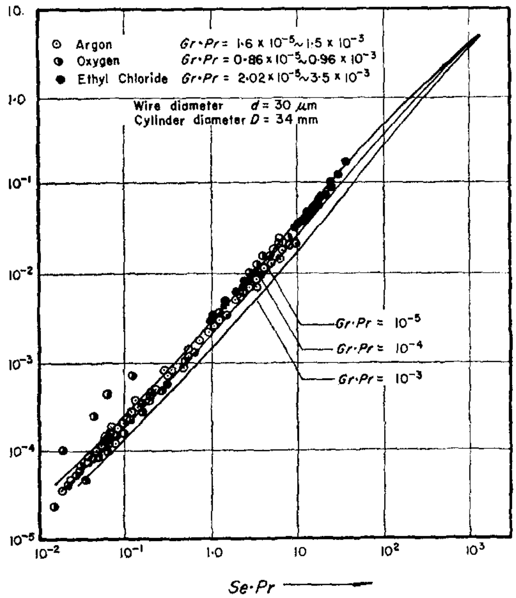

FIG. 4. The theoretical result for gases compared with the experiment of Senftleben and Braun [2].

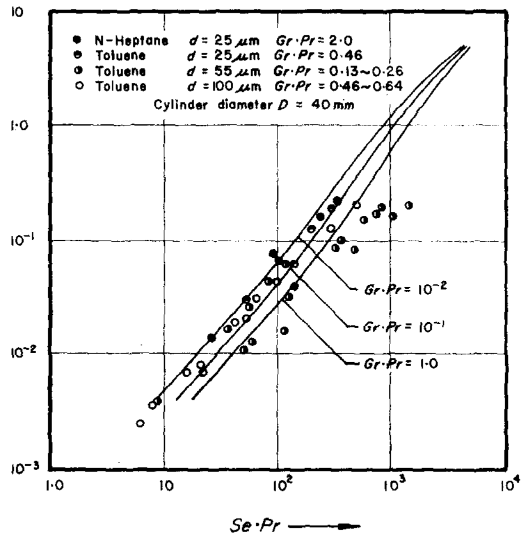

FIG. 5. The theoretical result for liquids compared with the experiment of Ashmann and Kronig [5].

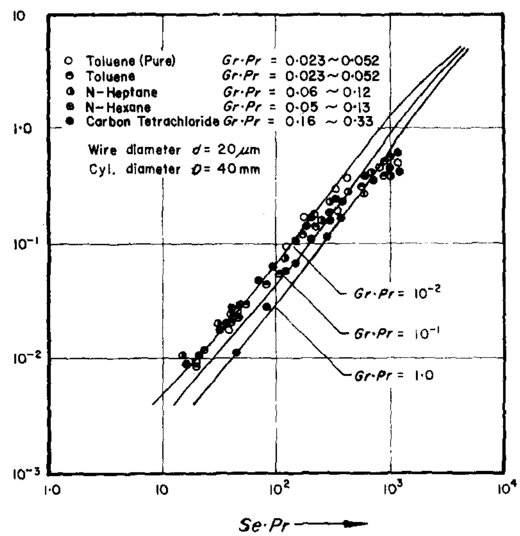

FIG. 6. The theoretical result for liquids compared with the experiment of De Haan [6].

It is seen that they are in very good agreement both in trend and magnitude. The circular points for oxygen falling away from the main bulk of experimental data are recognized in [4] to be in error. Figs. 5 and 6 show the experimental data of [5] and [6] compared with (16). Here again one experimental point in the *Nu* versus $(Se \cdot Pr)$ plane was used for a given Grashof number in order to evaluate C_4. The value $C_4 = 1{\cdot}0$ was chosen for both sets of data. For small values of the parameter $(Se \cdot Pr)$ the agreement with (16) is good, but for higher values there is a difference in trend upon comparison with the data for gases and the present theory. It is shown in [6] that these differences

† The constant is fixed by using *one and only one* experimental point corresponding to one $(Gr \cdot Pr)$ number and one $(Se \cdot Pr)$ number.

are due to the fact that the viscosity of liquids varies strongly with the temperature and that in the more extended field of flow corresponding to large diameters of the wire, some average value for the viscosity should be used rather than the value calculated for the temperature prevailing at the wall. On the other hand [5] gives as a possible explanation the fact that in their experiments the cylinder was closed at both ends thus impeding the circulation.

From Figs. 4, 5 and 6 we also see that the theory defines very well in terms of $(Gr \cdot Pr)$ the width of the band inside which the experimental points are contained. On the other hand from the scatter and range of these data, it is not possible to test the theoretical prediction which states that small differences in Nusselt numbers should correspond to higher Grashof numbers for the same Senftleben numbers.

4. CONCLUSIONS

When an electric field is applied between a horizontal heated wire and a coaxial cooled cylinder filled with an electrostrictive fluid, the heat-transfer rate from the wire to the cylinder increases; this is due to the presence of an additional thermal circulation current produced by the unbalanced electrostrictive forces in the upward direction. Since the temperature distribution after its modification by the gravitational convection is not axially symmetric with respect to the wire, the total electrostrictive force acting on the fluid located above the horizontal plane passing through the center of the wire, exceeds that found at the lower half and as a result, there will be a net force in the upward direction in addition to the gravitational one, thus producing an additional circulation and an increase in the heat transfer. An order of magnitude analysis based on nondimensional arguments and the fundamental conservation equations, combined with Langmuir's idea of a "conduction ring" for fine wires has yielded the following analytic expression for the variation of the Nusselt number with the parameters Gr, Pr, and Se.

$$Se \cdot Pr = \frac{C_5 - Gr \cdot Pr\,(e^{2/Nu} - 1)^4}{C_4\,(e^{2/Nu} - 1)}$$

where $C_4 = 5{\cdot}56$, $C_5\dagger = 546$ for gases and $C_4 = 1{\cdot}0$, $C_5 = 219$ for liquids. The above equation was tested with available experimental data and was found to be in good agreement with them.

ACKNOWLEDGEMENT

The authors wish to acknowledge with gratitude the financial assistance of the National Science Foundation through Grant No. G-22294.

REFERENCES

1. W. Panofsky and M. Philips, *Classical Electricity and Magnetism*. Addison–Wesley (1956).
2. H. Senftleben and W. Braun, Der Einfluss elektrischer Felder auf den Wärmestrom in Gasen, *Z. Phys.* **102**, 480–506 (1936).
3. J. E. Mayer and M. G. Mayer, *Statistical Mechanics*. Wiley, New York (1957).
4. R. Kronig and N. Schwarz, On the theory of heat transfer from a wire in an electric field, *Appl. Sci. Res.* A **1**, 35–46 (1949).
5. G. Ashmann and R. Kronig, The influence of electric fields on the convective heat transfer in liquids, *Appl. Sci. Res.* A **2**, 235–244 (1950). See also a correction to their data reported by them in *Appl. Sci. Res.* A **3**, 83–84 (1951).
6. H. J. De Haan, The influence of electric fields on the convective heat transfer in liquids II, *Appl. Sci. Res.* A **3**, 85–88 (1951).
7. I. Langmuir, Convection and conduction of heat in gases, *Phys. Rev.* **34**, 401 (1912).
8. A. Hermann, Wärmeübergang bei freier Strömung am waagerechten Zylinder in Zweiatomigen Gasen, Forschungschft379, Forsch. Gebiete Ingenieur (1936).
9. E. R. G. Eckert, *Introduction to the Transfer of Heat and Mass*. McGraw-Hill, New York (1950).
10. Boa-Teh Chu, Thermodynamics of electrically conducting fluids, *Phys. Fluids*, **2**, 5, 473–484 (1959).

APPENDIX A

Derivation of the Electrostrictive Force Term

The force on a neutral dielectric in the presence of a nonuniform electric field is [1]

$$\vec{f} = \frac{E^2}{2}\nabla\epsilon + \nabla\left(E^2 \frac{\partial\epsilon}{\partial\rho}\rho\right) \qquad \text{(A-1)}$$

† It should be noted again that the value C_5 is fixed by the solution of the non-electrostrictive problem, whereas the value C_4 is chosen for best fit *at one and only one* experimental point corresponding to a single set of the numbers Nu, $(Gr \cdot Pr)$, and $(Se \cdot Pr)$.

where ϵ is the dielectric constant. In the literature describing the phenomena in question, what is called electrostrictive force, is based on the force $\vec{f}$ which contains the component due to a non-uniform dielectric, besides the pure electrostrictive term [second contribution in (A-1)]. According to Debye's theory [3] the electric susceptibility χ for gases neglecting the Lorentz-Lorentz force is of the following form

$$\chi = \epsilon - 1 = \frac{N\rho}{M}\left\{\sigma + \frac{p_0^2}{3kT}\left[1 - \frac{1}{15}\left(\frac{p_0 E}{kT}\right)^2 + \dots\right]\right\} \simeq \frac{N\rho}{M}\left(\sigma + \frac{p_0^2}{3kT}\right) \quad \text{(A-2)}$$

where N = Avogadro's number
M = molecular weight of the gas
σ = coefficient of electric polarization
p_0 = permanent electric moment
k = Boltzmann's constant
T = absolute temperature.

In the last step of above equation all terms of order $[(p_0E)/(kT)]^2$ or higher have been neglected. This quantity represents the square of the non-dimensional ratio of the electrostatic energy of polarization to the molecular kinetic energy and is a very small number compared to one.†

From (A-2), it is easy to show that

$$\rho\frac{\partial\epsilon}{\partial\rho} = \rho\frac{\partial\chi}{\partial\rho} = \chi. \quad \text{(A-3)}$$

Combining (A-1) and (A-3) gives

$$\vec{f} = \frac{E^2}{2}\nabla\epsilon + \tfrac{1}{2}\nabla(E^2\chi) = \frac{\chi}{2}\nabla E^2. \quad \text{(A-4)}$$

If we denote by T_0, ρ_0 and $\vec{f}_0$ the temperature, density and the electrostrictive force in the absence of heating, the net force due to electrostriction is

$$\vec{f} - \vec{f}_0 = \tfrac{1}{2}(\chi - \chi_0)\nabla E^2. \quad \text{(A-5)}$$

For a small temperature difference $T - T_0$, substituting (A-2) into (A-5) yields

$$\vec{f} - \vec{f}_0 = -\tfrac{1}{2}\rho_0\beta\theta\gamma\nabla E^2 \quad \text{(A-6)}$$

where

$$\theta = T - T_0, \quad \beta = 1/T_0$$

and

$$\gamma = \frac{N}{M}\left(\sigma + \frac{2p_0^2}{3kT_0}\right).$$

For liquids, there is no explicit form for γ; Ashmann and Kronig [5] give the following expression

$$\gamma = \frac{1}{\rho_0\beta}\left(\frac{\partial\chi}{\partial T}\right)_p. \quad \text{(A-7)}$$

† For the experiments cited in the present report we have $p_0 \simeq 10^{-18}$ e.s.u. $T \simeq 300$°K, $E \simeq 10^5$ V/cm. We find $p_0E/kT \simeq 10^{-2}$.

APPENDIX B

The energy equation for fluids with magnetic permeability and dielectric constant depending on temperature and density

Boa-Teh Chu [10] has derived the energy equation for continuous media in the presence of electromagnetic fields as follows:

$$\rho\left[\frac{dU}{dt} + p\frac{d}{dt}\left(\frac{1}{\rho}\right) - \vec{H}\cdot\frac{d}{dt}\left(\frac{\vec{B}}{\rho}\right) - \vec{E}\cdot\frac{d}{dt}\left(\frac{\vec{D}}{\rho}\right)\right] = \nabla(\lambda\nabla T) + \phi. \quad \text{(B-1)}$$

In the above the function ϕ contains both the viscous and joulean dissipation. The internal energy U and pressure p are split into "mechanical" and "electromagnetic" parts as follows:

$$p = \underbrace{\rho RT}_{p_m} + \underbrace{\tfrac{1}{2}\left(\vec{B}\cdot\vec{H} - \rho H^2 \frac{\partial \mu_e}{\partial \rho}\right) + \tfrac{1}{2}\left(\vec{D}\cdot\vec{E} + \rho E^2 \frac{\partial \epsilon}{\partial \rho}\right)}_{p_e} \tag{B-2}$$

$$U = \underbrace{U_0 + \frac{R}{\gamma - 1}(T - T_0)}_{U_m} + \underbrace{\frac{1}{2\rho}\left[\frac{D^2}{\epsilon}\left(1 + \frac{T}{\epsilon}\frac{\partial \epsilon}{\partial T}\right) + \frac{B^2}{\mu_e}\left(1 + \frac{T}{\mu_e}\frac{\partial \mu_e}{\partial T}\right)\right]}_{U_e}. \tag{B-3}$$

The above are valid for isotropic fluids with constitutive equations of the form

$$\vec{D} = \epsilon(\rho, T)\vec{E}, \qquad \vec{B} = \mu_e(\rho, T)\vec{H}. \tag{B-4}$$

[10] states the theorem that *the energy equation reduces to its pure mechanical form, not only when* μ_e *and* ϵ *are constants, but also if they are at most functions of mass density but not of temperature.*

In this Appendix we show that, *if we carry the approximation of (A-2) (and the similar one for the magnetic permeability) into the energy equation, then within this approximation the energy equation reduces again to its pure mechanical form although both* μ_e *and* ϵ *are functions of temperature.* We present the proof only for the term involving the electric field since statements concerning the magnetic field terms are equivalent.

We use (A-2) in its approximate form to compute the partial derivatives in (B-2) and (B-3). From (B-2) it is easy to establish that

$$p = p_m + \frac{E^2}{2}. \tag{B-5}$$

From (B-3) we find

$$U = U_m + \frac{E^2}{2}\left[\frac{\epsilon}{\rho} - \frac{3kMT}{Np_0^2}\right]. \tag{B-6}$$

By using the above equations we find after some algebra:

$$\frac{\mathrm{d}U}{\mathrm{d}t} + p\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{1}{\rho}\right) - \vec{E}\cdot\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\epsilon\vec{E}}{\rho}\right)$$
$$= \frac{1}{2}\left[1 + \frac{2}{15}\left(\frac{p_0E}{kT}\right)^2\right]\frac{\mathrm{d}U_m}{\mathrm{d}t} + \frac{1}{2}\frac{\mathrm{d}}{\mathrm{d}t}$$
$$\left\{U_m\left[1 - \frac{2}{15}\left(\frac{p_0E}{kT}\right)^2\right]\right\} + p_m\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{1}{\rho}\right)$$
$$\simeq \frac{\mathrm{d}U_m}{\mathrm{d}t} + p_m\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{1}{\rho}\right). \tag{B-7}$$

In the last step we have neglected the quantity $(2/15)\,(p_0E/kT)^2$ as compared to one.

Using the above, the energy equation becomes,

$$\rho\left[\frac{\mathrm{d}U_m}{\mathrm{d}t} + p_m\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{1}{\rho}\right)\right] = \nabla\cdot(\lambda\nabla T) + \phi \tag{B-8}$$

and the equation contains only mechanical terms.

Assuming steady incompressible flow with negligible dissipation we find

$$\vec{q}\cdot\nabla T = \alpha\,\nabla^2 T \tag{B-9}$$

where α is the thermal diffusivity (B-9) and $\vec{q}$ is the velocity vector. This last equation is the one appropriate for the problem at hand.

Résumé—On étudie la convection libre d'un fluide "electrostrictif" à l'intérieur d'un cylindre refroidi, traversé suivant son axe par un fil chauffé, et soumis à un champ électrique radial non uniforme. On obtient une solution approchée par extension du modèle de conduction de Langmuir, pour les fils fins, au cas où il existe des forces d'électrostriction. On a trouvé que l'accroissement du nombre de Nusselt dû à la circulation supplémentaire produite par les forces d'électrostriction dépendait du nombre de Grashof, du nombre de Prandtl et d'un troisième nombre sans dimensions, de même nature que le nombre de Grashof, mais faisant intervenir le champ électrostatique au lieu du champ gravitationnel. Les résultats théoriques sont comparés aux données expérimentales, l'accord est satisfaisant.

Zusammenfassung—Die freie Konvektion in einer der Elektrostriktion unterworfenen Flüssigkeit wird in einem waagrecht liegenden, gekühlten Rohr, in dessen Achse ein Heizdraht verläuft und das einem nicht einheitlichen radialen Feld ausgesetzt ist, untersucht. Eine Näherungslösung liess sich durch Erweiterung des Langmuir'schen Leitungsmodells für dünne Drähte auf den elektrostriktiven Fall erhalten. Es zeigte sich, dass die Zunahme der Nusseltzahl infolge zusätzlicher, von den elektrostriktiven Kräften hervorgerufener Zirkulation von der Grashofzahl, der Prandtlzahl und einer dritten charakteristischen, dimensionslosen Grösse abhängt. Letztere ist von derselben Art wie die Grashofzahl, jedoch ist ihr eher ein elektrostatisches Feld als ein Schwerefeld zugrunde zu legen. Die theoretischen Ergebnisse werden mit verfügbaren experimentellen Daten verglichen und zeigen gute Übereinstimmung.

Аннотация—Исследуется свободная конвекция при течении электростриктивной жидкости между горизонтальной нагретой проволочкой и коаксиальным охлажденным цилиндром при применении неоднородного радиального электрического поля. Приближенное решение получено путем распространения модели условия Лангмюра для тонких проволочек к случаю электрострикации. Найдено, что увеличение числа Нуссельта за счет, дополнительной циркуляции, вызываемой электростриктивными силами, зависит от числа Грасгофа, числа Прандтля и третьего характеристичекого безразмерного числа типа числа Грасгофа, но отнесенного скорее к электростатическому полю, чем гравитационному. Теоретические результаты хорошо согласуются с имеющими экспериментальными данными.